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The isobaric-spin potential in heavy deformed nuclei

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Abstract. A very simple method for deriving the isobaric-spin potential is suggested in the present paper. It is based on using the proton and neutron occupation numbers in the framework of the single-particle level scheme. This technique is applied to the calculation of the symmetry energy parameter V_1 for thirty-five heavy deformed nuclei. The average value of V_1 is found to be 156 MeV, the deviations from this value for particular nuclei being small (for twenty-nine nuclei they do not exceed 10% of the average value, and for the rest they are less than 15%).

1. Introduction

In recent years the isobaric-spin potential has been intensively investigated (see, for example, the reviews by Hodgson 1964, Sood 1966) in the framework of the optical and shell models. The presence of this term is, apparently, the effect of the considerable contribution of the Heisenberg charge-exchange forces to the nucleon–nucleon interaction (Drell 1955). The isobaric-spin potential leads to a noticeable difference in the depth of the potentials for protons and neutrons (Lane 1962), but the most direct evidence for its presence is the 'quasi-elastic' (p, n) reaction (Lane 1962, Satchler *et al.* 1964). Furthermore, it manifests itself in the single-particle proton and neutron bound states and may be associated with the nuclear symmetry energy (Sood 1966).

However, although the existence of the isobaric-spin potential is now certain, its value is very unreliable, and different authors give widely differing results (Sood 1966, Green and Sood 1958, Rosen *et al.* 1965).

In this article we present a very simple method of obtaining the value of the symmetry energy parameter V_1. This method uses the single-particle level scheme and the proton and neutron occupation numbers of the scheme for the stable nuclei (occupation number method). The occupation number method, in spite of its extreme simplicity, gives stable values of the symmetry energy parameter V_1 for a large range of heavy nuclei.

2. The formulation of the method

For nuclei with Z protons and $N = A-Z$ neutrons the single-particle neutron and proton potentials may be written as follows:

$$U_n(r) = U_0(r) - \frac{A-1-2Z}{4(A-1)} U_1(r) \tag{1a}$$

$$U_p(r) = U_0(r) + \frac{A+1-2Z}{4(A-1)} U_1(r) - U_C(r) \tag{1b}$$

where $U_1(r)$ is the isobaric-spin potential and $U_C(r)$ is the Coulomb potential of the proton.

We shall also take into consideration the fact that $U_0(r)$ is the main part of the potential and assume that the isobaric-spin and the Coulomb potentials do not alter significantly the dependence of the total potential on r†. In this case the isobaric-spin and Coulomb

† This approximation (besides the relatively small contribution of $U_1(r)$ and $U_C(r)$) is justified for the following reasons. It is known that the neutron and proton density difference is approximately constant over the nuclear volume, which leads to the approximate constancy of $U_1(r)$ inside the nucleus. When considering $U_C(r)$ one should bear in mind the relative 'smoothness' of the Coulomb potential. We shall note here that, in principle, the relation

$$U_1(r) = \{U_p(r) - U_n(r) + U_C(r)\} \frac{2(A-1)}{A-2Z}$$

may be used to obtain $U_1(r)$, but the potentials $U_p(r)$ and $U_n(r)$ cannot be derived at present with sufficient accuracy for this purpose.

potentials will only produce corresponding displacements of the proton and neutron wells.

Considering

$$\begin{pmatrix} U_n(r) \\ U_p(r) \end{pmatrix} = \begin{pmatrix} V_n \\ V_p \end{pmatrix} f(r)$$

$$= \left\{ V_0 - t_3 \frac{A - 2t_3 - 2Z}{2(A-1)} V_1 - V_C(\tfrac{1}{2} - t_3) \right\} f(r) \qquad (2)$$

(the third component of the nucleon isobaric spin being designated by t_3) we obtain

$$V_1 = (\Delta V_{pn} + V_C) \frac{2(A-1)}{A-2Z} \qquad (3)$$

where $\Delta V_{pn} = V_p - V_n$ is the difference in the depths of the proton and neutron wells.

For the stable (against $\beta^{\pm}$ decay and K capture) nuclei the highest neutron and proton energies differ by less than 0·511 MeV. That is why the value of ΔV_{pn} may be obtained on the basis of the known single-particle level scheme as the energy difference between the highest filled proton and neutron levels, the scheme being filled with Z protons and $N = A - Z$ neutrons. The greater the difference $A - 2Z$, the smaller is the error in the method (i.e. the error is smallest in the region of the heavy nuclei; for nuclei with $A = 2Z$ the method is useless). The heavy nuclei usually being deformed, at present the Nilsson scheme is the only single-particle level scheme which is available in practice (Nilsson 1955)†. We shall use this scheme in practical calculations.

The proton Coulomb potential in the deformed axially-symmetric nuclei is (up to the first order in β) given by

$$U_C(r) \simeq \frac{(Z-1)e^2}{2R} \left[3 - \frac{r^2}{R^2} \{1 - 2\beta Y_{20}(\theta, \phi)\} \right] \qquad (4)$$

where the conventional notations are used. According to (2), only the displacement of the proton well caused by the Coulomb potential is significant. That is why we used the averaged value of $U_C(r)$ (averaged over the nuclear radius) as V_C, the small contribution of the deformed part of the Coulomb potential being neglected (this contribution did not exceed 5% even for strongly deformed nuclei). Then we have

$$V_C \simeq 1{\cdot}55 \frac{Z-1}{A^{1/3}} \text{ MeV}; \qquad (5)$$

the nuclear radius was taken as $R = r_0 A^{1/3}$; $r_0 = 1{\cdot}22$ fm.

3. Calculations and discussion

The occupation number method was applied to obtain the symmetry energy parameter V_1 in the case of heavy deformed even–even nuclei in the regions $150 \leqslant A \leqslant 194$ and $222 \leqslant A \leqslant 238$. The deformation parameters were taken from the review by Davidson (1965). The Nilsson scheme for the appropriate deformation was filled with Z protons and $N = A - Z$ neutrons and the value ΔV_{pn} was thus obtained. The main error in the calculations carried out results from the inaccuracy of the graphical determination of ΔV_{pn}. (To avoid extrapolating the Nilsson scheme we limited ourselves to nuclei with $\beta \leqslant 0{\cdot}31$.)

The results of the calculations are summarized in table 1.

The values of V_1 show satisfactory consistency among themselves and with the results of previous determinations; these values do not differ from each other significantly in spite of the wide range of nuclei considered. The average magnitude of V_1 was found to be equal to 156 MeV.

† Recently some articles have appeared where the deformed potentials of the Woods–Saxon type have been investigated (Faessler and Sheline 1966). However, calculations have been carried out so far only for selected nuclei.

Table 1. The symmetry parameters V_1 of the isobaric-spin potential for the heavy deformed nuclei

No.	Nucleus	β	ΔV_{pn} (MeV)	V_C (MeV)	V_1 (MeV)	No.	Nucleus	β	ΔV_{pn} (MeV)	V_C (MeV)	V_1 (MeV)
1	^{150}Sm	0·184	−4·35	18·0	156	19	^{190}Os	0·180	−5·10	20·1	149
2	^{152}Sm	0·290	−3·64	17·7	152	20	^{192}Os	0·160	−5·27	20·1	141
3	^{154}Gd	0·280	−3·47	18·3	174	21	^{194}Pt	0·152	−5·23	20·6	156
4	^{160}Dy	0·301	−4·02	18·6	166	22	^{222}Ra	0·184	−5·47	22·2	160
5	^{164}Er	0·306	−3·58	18·9	178	23	^{224}Ra	0·171	−5·50	22·2	155
6	^{170}Er	0·310	−3·68	18·8	150	24	^{226}Ra	0·197	−5·97	22·2	146
7	^{170}Yb	0·304	−3·96	19·2	171	25	^{228}Ra	0·212	−6·19	22·0	138
8	^{172}Yb	0·311	−4·10	19·1	160	26	^{226}Th	0·220	−5·70	22·7	166
9	^{174}Yb	0·308	−4·56	19·1	148	27	^{228}Th	0·225	−5·60	22·5	160
10	^{176}Yb	0·301	−4·73	18·9	138	28	^{230}Th	0·233	−5·80	22·5	153
11	^{176}Hf	0·300	−4·18	19·5	167	29	^{232}Th	0·243	−6·03	22·4	146
12	^{178}Hf	0·310	−4·35	19·5	158	30	^{234}Th	0·233	−5·95	22·4	141
13	^{180}Hf	0·270	−4·93	19·4	144	31	^{230}U	0·245	−5·38	23·0	175
14	^{182}W	0·280	−4·48	19·9	164	32	^{232}U	0·257	−5·52	23·0	168
15	^{184}W	0·250	−5·06	19·9	150	33	^{234}U	0·251	−5·62	22·8	160
16	^{186}W	0·259	−5·62	19·9	139	34	^{236}U	0·263	−6·70	22·8	145
17	^{186}Os	0·200	−4·68	20·3	170	35	^{238}Pu	0·271	−6·53	23·1	157
18	^{188}Os	0·191	−4·88	20·3	158						

In a more refined single-particle level scheme, the average value of V_1 would be slightly different. The stability of the results obtained demonstrates, however, the fact that in the case considered the Nilsson scheme is a sufficiently perfect tool for dealing with heavy deformed nuclei.

It should be noted that the value of V_1 decreases for the isotopes of a given element when the number of neutrons increases; this is true for all the nuclei considered. At the same time, as is seen from the table (see, for instance, the data for Yb, W, Th), the dependence of V_1 upon A for a given Z is nearly linear† with

$$-\frac{\partial V_1}{\partial A} \simeq (3{\cdot}1 - 5{\cdot}7) \text{ MeV}.$$

However, one should bear in mind that in our case, besides real instability of V_1, the effect of the decrease in V_1 with increase in A is partially connected with the assumption (made when ΔV_{pn} was calculated) concerning the equality of the energies of the 'highest' neutron and proton levels. As we have already mentioned, this assumption is valid in the framework of the single-particle scheme only with an accuracy of 0·511 MeV for stable nuclei (for the β^--active nuclei ^{228}Ra, ^{232}Th, ^{234}Th there is less accuracy). We shall limit ourselves here to this short remark concerning the dependence of V_1 on A, as for a detailed analysis one should use a more refined single-particle level scheme (instead of the Nilsson scheme) taking the residual nucleon interactions into account.

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† The fluctuations from linear dependence may be partially caused by the coarseness of the assumption made as to the value of r_0 (1·22 fm) for all the nuclei considered.